9. MAGNETOHYDRODYNAMIC (MHD) POWER GENERATION

When an electrical conductor is moved so as to cut lines of magnetic induction, the charged particles in the conductor experience a force in a direction mutually perpendicular to the $B$ field and to the velocity of the conductor. The negative charges tend to move in one direction, and the positive charges in the opposite direction. This induced electric field, or motional emf, provides the basis for converting mechanical energy into electrical energy. At the present time nearly all electrical power generators utilize a solid conductor which is caused to rotate between the poles of a magnet. In the case of hydroelectric generators, the energy required to maintain the rotation is supplied by the gravitational motion of river water. Turbogenerators, on the other hand, generally operate using a high-speed flow of steam or other gas. The heat source required to produce the high-speed gas flow may be supplied by the combustion of a fossil fuel or by a nuclear reactor (either fission or possibly fusion).

It was recognized by Faraday as early as 1831 that one could employ a fluid conductor as the working substance in a power generator. To test this concept Faraday immersed electrodes into the Thames river at either end of the Waterloo Bridge in London and connected the electrodes at mid span on the bridge through a galvanometer. Faraday reasoned that the electrically conducting river water moving through the earth's magnetic field should produce a transverse emf. Small irregular deflections of the galvanometer were in fact observed. The production of electrical power through the use of a conducting fluid moving through a magnetic field is referred to as magnetohydrodynamic, or MHD, power generation. One of the earliest serious attempts to construct an experimental MHD generator was undertaken at the Westinghouse laboratories in the period 1938-1944, under the guidance of Karlovitz (see Karlovitz and Halász, 1964). This generator (which was of the annular Hall type—see Fig. 20) utilized the products of combustion of natural gas, as a working fluid, and electron beam ionization. The experiments did not produce the expected power levels because of the low electrical conductivity of the gas and the lack of existing knowledge of plasma properties at that time. A later experiment at Westinghouse by Way, DeCorso, Hundstad, Kemeny, Stewart, and Young (1961), utilizing a liquid fossil fuel "seeded" with a potassium compound, was much more successful and yielded power levels in excess of 10 kW. Similar power levels were achieved at the Avco Everett laboratories by Rosa (1961) using arc-heated argon at 3000°C "seeded" with powdered potassium carbonate. In these latter experiments "seeding" the working gas with small concentrations of potassium was essential to provide the necessary number of free electrons.
required for an adequate electrical conductivity. (Other possible seeding materials having a relatively low ionization potential are the alkali metals cesium or rubidium.)

During the decade beginning about 1960 three general types of MHD generator systems evolved, classified according to the working fluid and the anticipated heat source. *Open-cycle* MHD generators operating with the products of combustion of a fossil fuel are closest to practical realization. In the United States, operation of a 32 MW alcohol-fueled generator with run times up to three minutes was achieved in 1965 (see Mattsson, Ducharme, Govoni, Morrow, and Brogan, 1965). In the Soviet Union tests on a 75 MW (25 MW from MHD and 50 MW from steam) pilot plant burning natural gas began in 1971. *Closed-cycle* MHD generators are usually envisaged as operating with nuclear reactor heat sources, although fossil fuel heat sources have also been considered. The working fluid for a closed-cycle system can be either a seeded noble gas or a liquid metal. Because of temperature limitations imposed by the nuclear fuel materials used in reactors, closed-cycle MHD generators utilizing a gas will require that the generator operate in a nonequilibrium mode. We shall have more to say later about some of the difficulties that nonequilibrium operation entails. The subject of liquid metal MHD generators lies outside the scope of our discussion.

An MHD generator, like a turbogenerator, is an energy conversion device and can be used with any high-temperature heat source—chemical, nuclear, solar, etc. The future electrical power needs of industrial countries will have to be met for the most part by thermal systems composed of a heat source and an energy conversion device. In accordance with thermodynamic considerations, the maximum potential efficiency of such a system (i.e., the Carnot efficiency) is determined by the temperature of the heat source. However, the maximum actual efficiency of the system will be limited by the maximum temperature employed in the energy conversion device. The closer the temperature of the working fluid in the energy conversion device to the temperature of the heat source, the higher the maximum potential efficiency of the overall system. A spectrum of heat source temperatures are currently available, up to about 3000°K. However, at the present time large central-station power production is limited to the use of a single energy-conversion scheme—the steam turbogenerator—which is capable of operating economically at a maximum temperature of only 850°K. The over-all efficiencies of present central-station power-producing systems are limited by this fact to values below about 42 percent, which is a fraction of the potential efficiency. It is clear that a temperature gap exists in our energy conversion technology.

Because MHD power generators, in contrast to turbines, do not require
the use of moving solid materials in the gas stream, they can operate at much higher temperatures. Calculations show that fossil-fueled MHD generators may be capable of operating at efficiencies between 50 and 60 percent. Higher operating efficiencies would lead to improved conservation of natural resources, reduced thermal pollution, and lower fuel costs. Studies currently in progress suggest also the possibility of reduced air pollution. In this section an elementary account of some of the concepts involved in MHD power generation is presented. A more complete discussion may be found in the book by Rosa (1968).

The essential elements of a simplified MHD generator are shown in Fig. 15. This type of generator is referred to as a continuous electrode Faraday generator. A field of magnetic induction $B$ is applied transverse to the

![Figure 15. A simplified MHD generator.](image)
motion of an electrically conducting gas flowing in an insulated duct with a velocity $u$. Charged particles moving with the gas will experience an induced electric field $u \times B$ which will tend to drive an electric current in the direction perpendicular to both $u$ and $B$. This current is collected by a pair of electrodes on opposite sides of the duct in contact with the gas and connected externally through a load. Neglecting the Hall effect, the magnitude of the current density for a weakly ionized gas is given by the generalized Ohm’s law (8.26) as

$$J = \sigma(E + u \times B).$$  \hspace{1cm} (9.1a)

The electric field $E$, which is added to the induced field, results from the potential difference between the electrodes. For the purposes of our initial discussion in this section we shall assume that both $u$ and $\sigma$ are uniform.

In terms of the coordinate system shown in Fig. 15, we have that

$$J_y = \sigma(E_y - uB).$$  \hspace{1cm} (9.1b)

At open circuit $J_y = 0$, and so the open circuit electric field is $uB$ [cf. equation (7.3)]. For the characteristic conditions $u \sim 1000 \text{ m sec}^{-1}$ and $B \sim 2 \text{ T}$, the open circuit electric field is $uB \sim 2000 \text{ V m}^{-1}$. At short circuit $E_y = 0$, and the short circuit current is $J_y = -\sigma uB$. For general load conditions, it is conventional to introduce the loading parameter

$$K \equiv \frac{E_y}{uB},$$  \hspace{1cm} (9.2)

where $0 \leq K \leq 1$, and write $J_y = -\sigma uB(1 - K)$. The negative sign indicates that the conventional current flows in the negative $y$-direction. Since the electrons flow in the opposite direction, the bottom electrode must serve as an electron emitter, or cathode, and the upper electrode is an anode.

In accordance with equation (4.5a), the electrical power delivered to the load per unit volume of a MHD generator gas is

$$P = -J \cdot E.$$  \hspace{1cm} (9.3a)

For the generator shown in Fig. 15,

$$P = \sigma u^2 B^2 K(1 - K).$$  \hspace{1cm} (9.3b)

This power density has a maximum value

$$P_{\text{max}} = \frac{\sigma u^2 B^2}{4},$$  \hspace{1cm} (9.4)

for $K = 1/2$. In accordance with equation (4.4), the rate at which directed energy is extracted from the gas by the electromagnetic field per unit
volume is \(-\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B})\). We therefore define the electrical efficiency of a MHD generator as

\[ \eta_e = \frac{\mathbf{J} \cdot \mathbf{E}}{\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B})}. \]  

(9.5a)

For the generator being discussed,

\[ \eta_e = K. \]  

(9.5b)

The Faraday generator therefore tends to higher efficiency near open circuit operation.

In order that a MHD generator have an acceptable size, it is necessary that the generator deliver a minimum of about 10 MW per cubic meter of gas. Using the preceding characteristic values for \(u\) and \(B\), this requirement means that the electrical conductivity must be such that

\[ \sigma \gtrsim \frac{4 P_{\text{max}}}{u^2 B^2} \sim 10 \text{ mhos m}^{-1}. \]  

(9.6)

![Figure 16](image)

**Figure 16.** Representative values of the electrical conductivity of MHD generator plasmas at 1 atm (—— products of combustion of \(C_2H_5OH\) + \(3O_2\) with 0.5 mass fraction \(N_2/O_2\), seeded with 0.01 mass fraction \(K\); ——— argon seeded with 0.004 mole fraction \(K\); ——— argon seeded with 0.004 mole fraction \(Cs\)).
Current densities will then be of the order of a few amperes per square cm or more. The equilibrium electrical conductivities at atmospheric pressure of a potassium-seeded combustion products plasma, and potassium-seeded and cesium-seeded argon plasmas are shown in Fig. 16, plotted as a function of temperature. The gas temperatures needed to achieve the condition (9.6) can be readily obtained with many fossil fuels. However, exit gas temperatures available from present-day nuclear reactors are too low, and it is necessary to examine the feasibility of employing nonequilibrium methods to obtain enhanced values of the electrical conductivity.

Shown in Fig. 17 are the values of the electron Hall parameter for a magnetic induction of 1 T, corresponding to the gases and conditions described in Fig. 16. (To a good approximation, the Hall parameter scales linearly with $B$.) It is apparent that the Hall effect can play a significant role in the operation of an MHD generator, and it is necessary to review the preceding analysis. Because of the Hall effect, a current flowing in the y-direction can give rise to a current flowing in the x-direction. Instead

![Figure 17. Representative values of the electron Hall parameter of MHD generator plasmas for $B = 1.0$ T at 1 atm (—— products of combustion of $C_2H_5OH + 3O_2$ with 0.5 mass fraction $N_2/O_2$, seeded with 0.01 mass fraction K; --- argon seeded with 0.004 mole fraction K; -- -- argon seeded with 0.004 mole fraction Cs).]
of the Ohm's law in the form of equation (9.1), we must now write [cf. equations (8.26), (3.21), and (3.11)]

\[ J_y = \frac{\sigma}{1 + \beta^2} (E_y + \beta E_x), \quad (9.7a) \]

and

\[ J_x = \frac{\sigma}{1 + \beta^2} (E_x - \beta E_y). \quad (9.7b) \]

(For simplicity, we shall employ the notation \( \beta = \beta_e \) henceforth.) Because the electrodes in a generator of the type shown in Fig. 15 extend the entire length of the duct, they tend to impose equipotential surfaces in the gas which are normal to the y-direction. Thus for a continuous electrode Faraday generator

\[ E_x = 0. \quad (9.8a) \]

It follows from equations (9.7) that

\[ J_y = \frac{\sigma}{1 + \beta^2} E_y = \frac{-\sigma u B(1 - K)}{1 + \beta^2}, \quad (9.8b) \]

\[ J_x = \frac{-\beta \sigma}{1 + \beta^2} E_y = -\beta J_y, \quad (9.8c) \]

and from equation (9.3a) that

\[ P = \frac{\sigma u^2 B^2}{1 + \beta^2} K(1 - K). \quad (9.8d) \]

The Hall effect reduces \( J_y \) and \( P \) by the factor \( (1 + \beta^2) \) and results in the appearance of a Hall current which flows downstream in the gas and returns upstream through the electrodes. The reduction of \( J_y \) and \( P \) is caused by the fact that the \( uB \) field must overcome not only the \( E_x \) field produced by the electrodes, but also the Hall emf resulting from the current flow in the x-direction.

To circumvent the deleterious consequences of the Hall effect, the electrodes may be segmented in the manner indicated in Fig. 18b and separate loads connected between opposed electrode pairs. In the limit of infinitely fine segmentation, there can be no \( x \)-component of current either in the electrodes or in the gas, and so the condition for an ideal segmented Faraday generator is

\[ J_x = 0. \quad (9.9a) \]
From equation (9.7b) we see that this configuration leads to the build-up of an electric field in the $x$-direction, the so-called Hall field

$$E_x = \beta E'_y.$$  \hfill (9.9b)

With this value for $E_x$, equation (9.7a) becomes

$$J_y = \sigma E'_y,$$  \hfill (9.9c)

which is identical to the relation (9.1b) that we used when we neglected the Hall effect. We also recover the value for the power density in the
absence of the Hall effect given by equation (9.3b). In an actual generator with finite segmentation, the Hall effect causes the current to concentrate at the upstream edge of an anode and at the downstream edge of a cathode (see Rosa, 1968, p. 68). Thus the Hall currents are not completely suppressed, and the improvement in performance of a segmented Faraday generator is not as good as the foregoing idealized calculation would suggest. An experimental power curve for a combustion products MHD generator obtained by Way, De Corso, Hunstad, Kemeny, and Stewart (1961) is shown in comparison with theory, in Fig. 19. The discrepancy is attributed in part to leakage currents in the generator walls.

![Figure 19. Electric power produced by an experimental combustion products MHD generator compared with theory (after Way, DeCorso, Hunstad, Kemeny, Stewart, and Young, 1961).](image)

The action of the Hall effect in building up an axial electric field suggests that this field can be used to drive a load connected between the upstream and downstream electrodes. According to equation (9.9b), the magnitude of the axial electric field will be a maximum when the opposed electrodes are shorted, i.e.,

$$E_y = 0.$$  \hspace{1cm} (9.10a)

In principle the shorting may be accomplished either externally or internally, as indicated in Fig. 18c. This type of connection is referred to as a Hall generator. For the condition (9.10a) the Ohm's law equations (9.7) become

$$J_y = \frac{\sigma}{1 + \beta^2}(-uB + \beta E_x),$$ \hspace{1cm} (9.10b)

and

$$J_x = \frac{\sigma}{1 + \beta^2}(E_x + \beta uB).$$ \hspace{1cm} (9.10c)
The open-circuit electric field for a Hall generator is obtained from equation (9.10c) as $-\beta uB$. We may therefore define the loading parameter for a Hall generator to be

$$K_H \equiv \frac{-E_x}{\beta uB},$$

(9.10d)

and rewrite the Ohm's law equations in the form

$$J_y = -\sigma uB \frac{(1 + \beta^2 K_H)}{1 + \beta^2},$$

(9.10e)

and

$$J_x = \sigma uB \frac{\beta}{1 + \beta^2} (1 - K_H).$$

(9.10f)

For a Hall generator, the power density is

$$P = -J_x E_x = \sigma u^2 B^2 \frac{\beta^2}{1 + \beta^2} K_H (1 - K_H),$$

(9.10g)

and the electrical efficiency is

$$\eta_e = \frac{J_x E_x}{J_y uB} = \frac{\beta^2 K_H (1 - K_H)}{1 + \beta^2 K_H}.$$

(9.10h)

For large Hall parameters, the power density of a Hall generator approaches that of a segmented Faraday generator. In contrast to segmented Faraday generators, Hall generators tend to have higher electrical efficiencies near (but not at) short circuit and offer the simplicity of a two-terminal connection.

The continuous Faraday and Hall generators may be viewed as special cases of a class of two-terminal generators referred to as diagonal conducting-wall generators. As illustrated in Fig. 18d, the diagonal conductors, which form part of the duct, are in contact with the plasma and tend to impose diagonal equipotential surfaces in the plasma. Thus, the condition defining a particular diagonal conducting wall generator is

$$\frac{E_y}{E_x} = -\tan \alpha,$$

(9.11)

where $\alpha$ is the angle between the plane passing through a conductor and the $y$-$z$ plane. The Hall and continuous Faraday generators correspond respectively to selecting $\alpha$ as either 0 or $\pi/2$. For further discussion of these generators, the reader is referred to the book by Rosa (1968).

In addition to the linear geometry and its various electrode configurations, MHD generators having other geometries have also been proposed. In
the disk and annular Hall generators shown in Fig. 20, the current component induced to flow in the $u \times B$ direction closes upon itself within the plasma, thereby obviating the need for segmented electrodes. The vortex geometry is similar to a continuous electrode Faraday generator and thus requires that the Hall parameter be small.

It has been pointed out by Rosa (1962) that nonuniform plasma properties in high Hall parameter devices can cause large degradations in performance. To show this effect let us consider a MHD duct with either continuous or finely segmented electrode walls, in which $\sigma$ and $\beta$ depend only on the coordinate $y$. Such a nonuniformity could result, for example, from wall-cooling, from insufficient seed-mixing, from nonuniform combustion, or from nonequilibrium effects. Let us assume that a steady-state solution of the electrodynamic equations exists where $J$ and $E'$ also depend on the $y$-coordinate only. It then follows from the equations $V \cdot J = 0$ and $V \times E = 0$ [cf. equations (6.10e) and (6.10f)], that

$$J_y = \text{constant}, \quad E_x = \text{constant}. \quad (9.12)$$

The overall performance of the device will depend on the values of $J_x(y)$ and $E'_y(y)$ averaged over the $y$ dimension of the duct. Solving the Ohm's law equations (9.7) for the latter quantities in terms of the former and averaging the result, we obtain

$$\bar{E}_y = \left(\frac{1 + \beta^2}{\sigma}\right)J_y - \bar{\beta}E_x, \quad (9.13a)$$

$$\bar{J}_x = \bar{\sigma}E_x - \bar{\beta}J_y. \quad (9.13b)$$

If $h$ denotes the $y$-dimension of the duct, then for any quantity $f(y)$, $\bar{f} = h^{-1} \int_0^h f(y) \, dy$. Equations (9.13) express the averaged field quantities in terms of averaged plasma properties. The simplicity of this result stems from the assumed one dimensionality of the nonuniformity.

If we recast equations (9.13) into the form of the original Ohm's law equations, we obtain

$$\left(\frac{1 + \beta^2}{\sigma}\right)J_y = \bar{E}_y + \bar{\beta}E_x, \quad (9.14a)$$

$$\left(\frac{1 + \beta^2}{\sigma}\right)\bar{J}_x = GE_x - \bar{\beta}\bar{E}_y. \quad (9.14b)$$

The nonuniformity factor

$$G = \bar{\sigma}\left(\frac{1 + \beta^2}{\sigma}\right) - \bar{\beta}^2 \quad (9.15a)$$
Figure 20. MHD generator geometries.
contains the coupling of nonuniformities and the Hall effect and is equal to unity for a uniform gas. If the Hall parameter is uniform,

$$G = 1 + (1 + \beta^2)(\sigma \sigma^{-1} - 1). \quad (9.15b)$$

This expression shows how the effect of even a small departure from a uniform conductivity becomes amplified at large Hall parameters. For an ideal segmented Faraday generator, we obtain in place of equations (9.9b) and (9.3b),

$$E_x = \frac{\beta E_y}{G}, \quad (9.16a)$$

and

$$\overline{P} = \frac{\sigma u^2 B^2 \overline{K}(1 - \overline{K})}{G}, \quad (9.16b)$$

where $\overline{K} = \overline{E}_y/(\overline{u}B)$. Thus, both the Hall field and the power density are reduced by the factor $1/G$. The dependence of $1/G$ on $\beta$ and on the degree of nonuniformity is shown in Fig. 21 for a particular distribution of $\sigma(y)$.

The elevation of electron temperature in a uniform and steady discharge in a noble gas seeded with an alkali metal is given by the electron energy equation [cf. equation (5.4)] as

$$J \cdot \mathbf{E}' = 3n_e k(T_e - T) \frac{m_e}{m_H} \overline{v}_{eH} + \dot{\mathcal{R}}. \quad (9.17)$$

![Figure 21. Effect of a nonuniform conductivity profile on the power density of a segmented Faraday MHD generator.](image_url)
Here $m_H$ is a weighted average of the masses of the heavy particles, $T$ is the heavy-particle gas temperature, $\dot{R}$ is the local net rate of radiation energy loss per unit volume, and we have employed the approximation $J_e \approx J$. For a discharge with an applied electric field and zero $B$ field, the left-hand side of this equation can be written $J^2/\sigma$. If we assume that $n_e$ and $T_e$ are related by Saha's equation [cf. equations (II 10.5)], and if we use the relation $\sigma = n_e e^2/m_e \bar{v}_{eH}$, then equation (9.17) may be used to predict the dependence of $\sigma$ on the current density $J$. Experiments to test this model at atmospheric pressures and high gas temperatures were first undertaken by Kerrebrock (1962). Some representative experimental results and a comparison with theory obtained by Cool and Zukoski (1966) are shown in Fig. 22 for potassium-seeded atmospheric pressure argon. The interpretation of the data at low current densities is complicated because of frozen-flow and other nonequilibrium effects, but at current densities of practical interest agreement between theory and experiment is quite satisfactory. These experiments show that the electrical conductivity can be significantly increased by nonequilibrium ionization.

For a MHD generator, the left-hand side of equation (9.17) may be evaluated using the Ohm's law relations (9.7). Neglecting the radiation loss term, we obtain

$$3 n_e k T \left( \frac{T_e}{T} - 1 \right) \frac{m_e}{m_H} \bar{v}_{eH} = \frac{\sigma E_e^2}{1 + \beta^2}.$$  \hspace{1cm} (9.18)

The value of $E_e^2$ depends on the type of MHD generator being considered. Shown in Table 2 are expressions for the ratio of the electron temperature to the gas stagnation temperature

$$T_e = T \left[ 1 + \frac{(\gamma - 1)}{2} M^2 \right],$$

for each of the three main types of linear MHD generator previously discussed. Here $\gamma$ is the ratio of specific heats of the gas, $\dot{\gamma} = (R m_H/k) \gamma \approx \gamma$, and

$$M^2 = \frac{u^2}{\gamma RT}$$

is the square of the Mach number of the gas. To make more evident the dependence on the Hall parameter, the last line of Table 2 shows the limiting values of $T_e/T_0$ for $\dot{\gamma} = \gamma = 5/3$, large Mach number, and optimal loading. The maximum elevation in electron temperature for a continuous Faraday generator is of the order of the gas stagnation temperature. However, for
Figure 22. Nonequilibrium ionization in atmospheric pressure potassium-seeded argon (after Cool and Zukoski, 1966).
Table 2  Magnetically Induced Elevation in Electron Temperature for Closed Cycle Linear MHD Generators

<table>
<thead>
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<th>Faraday</th>
<th>Cont.</th>
<th>Seg.</th>
<th>Hall</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>$E_x'$</td>
<td>$E_y'$</td>
<td>$K_H\beta uB$</td>
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<td>$T_e/T_0$</td>
<td></td>
<td>$1 + \frac{\gamma(1-K)^2}{3} \frac{\beta^2}{1+\beta^2} M^2$</td>
<td>$1 + \frac{\gamma(1-K)^2}{3} \frac{\beta^2}{1+\beta^2} M^2$</td>
<td>$1 + \frac{\gamma(1+\beta^2 K_H^2)}{3} \frac{\beta^2}{1+\beta^2} M^2$</td>
</tr>
<tr>
<td>$M \gg 1$</td>
<td>$\gamma = \frac{5}{3}$</td>
<td>$\frac{5}{3} \frac{\beta^2}{1+\beta^2}; K \to 0$</td>
<td>$\frac{5}{3} \beta^2; K \to 0$</td>
<td>$\frac{5}{3} \left(\frac{\beta^2}{1+\beta^2}\right)$ $K_H \to 1$</td>
</tr>
</tbody>
</table>

the segmented Faraday and Hall generators, this theory predicts that considerably higher electron temperatures are possible at high values of the Hall parameter.

The enhanced electrical conductivities implied by the preceding theory have not been experimentally observed. The reasons are not completely understood at the present time, but it is generally believed that a major contributing factor is that the plasma becomes unstable and that fluctuating inhomogeneities in the plasma properties develop. We shall discuss the origin of this instability in the next section.

It has been suggested by Solbes and Kerrebrock (1968) that for a Faraday generator, one should write in place of the expressions in Table 2, the relation

$$\frac{T_e}{T} - 1 = \frac{\gamma}{3} M^2 (1-K)^2 \beta^2 \frac{\sigma_{\text{eff}}}{\sigma} \frac{1 + \beta_{\text{app}}^2}{1 + \beta_{\text{eff}}^2}. \quad (9.19)$$

The bar here denotes a spatial average; the effective Hall parameter $\beta_{\text{eff}}$ is defined as the tangent of the angle between $\vec{J}$ and $\vec{E}'$ (see Fig. 9); the effective conductivity is defined by the relation $\sigma_{\text{eff}} = \vec{J}/\vec{E}'$, where $\vec{E}'_j$ is the average of the projection of $\vec{E}'$ on $\vec{J}$; the apparent Hall parameter is defined by the equation $\beta_{\text{app}} = \vec{E}'_x [uB(1-K)]$. For a uniform plasma equation (9.19) reduces to

$$\frac{T_e}{T} - 1 = \frac{\gamma}{3} M^2 (1-K)^2 \beta^2 \frac{1 + \beta_{\text{app}}^2}{1 + \beta^2},$$
from which one may see the extreme sensitivity, for large $\beta$, of $T_e/T$ to the Hall voltage recovery $\beta_{app}/\beta$. For an unstable plasma $\beta_{eff}$ is less than $\beta$, and appears to saturate at a value of order unity for large $\beta$, while $\sigma_{eff}$ appears (in some experiments) to approach $\beta_{eff}/\beta$. Thus, for an unstable plasma $(T_e/T) - 1$ varies as $\beta$ for large $\beta$. This result should be contrasted with the behavior of a stable plasma for large $\beta$, where $(T_e/T) - 1$ varies as $\beta^2$ for segmented electrodes, but is independent of $\beta$ for continuous electrodes. It would appear on the basis of expression (9.19) that nonequilibrium operation of a Faraday generator should be possible even in the presence of instabilities and that the level of nonequilibrium should increase with increasing magnetic field, albeit not as strongly as predicted for an ideally segmented Faraday generator.

**Exercise 9.1.** For a Hall generator, show that the loading parameter for maximum electrical efficiency is $K_H = (\sqrt{1 + \beta^2} - 1)/\beta^2$, and that the maximum electrical efficiency is $\eta_e = 1 - 2(\sqrt{1 + \beta^2} - 1)/\beta^2$.

**Exercise 9.2.** Discuss the performance features of a diagonal conducting wall generator.

**Exercise 9.3.** Discuss the effects of one-dimensional nonuniformities on the performance of a Hall generator.

**Exercise 9.4.** Compare the effects of ion slip on the maximum power outputs of the three main types of MHD generator electrode connections.