1. INTRODUCTION

We wish to turn now to a consideration of the effects of magnetic fields on the behavior of partially ionized gases. Our discussion will be based on the fundamental (nonrelativistic) expression

\[ F = q[E(x, t) + w \times B(x, t)] \quad (1.1) \]

for the force exerted on a particle of charge \( q \) and velocity \( w = dx/dt \), by an electric field \( E(x, t) \) and a magnetic induction field \( B(x, t) \). The validity of this relation, often referred to as the Lorentz force, may be viewed as justified by experiment. We shall discuss first several aspects of the motion of a single particle as governed by equation (1.1). We shall then discuss the effects of collisions and the consequences of equation (1.1) on a macroscopic level.

The generalization of the continuum fluid equations to provide a description of collision-dominated conducting gases leads to a closed set of fluid equations called the MHD equations, and we shall show how these may be obtained. In many situations, it is necessary to regard the electrons as a separate fluid coexisting with a fluid made up of heavy particles. For such cases it is necessary to supplement the usual fluid equations with an energy equation for the electrons. We shall also discuss the relation, referred to as a "generalized Ohm's Law," which describes how a current flows in a gas in response to electric and magnetic fields. Examples of some applications of these equations, presented in the latter part of this chapter, include discussions of Hartmann flow, MHD power generation, and the two-temperature ionization instability.

Equation (1.1) describes the force on a moving charged particle in any inertial frame and in particular applies to a particle having the velocity \( w \) in the laboratory reference frame \( S \). As illustrated in Fig. 1, with respect to a reference frame \( S' \) having a constant velocity \( w \) relative to \( S \), the particle...
would appear instantaneously at rest. In terms of quantities defined in the $S'$ frame, the force on the particle would be

$$F' = q'E'. \quad (1.2)$$

Within the context of nonrelativistic mechanics, force and charge transform according to the relations $F' = F$ and $q' = q$. Comparing equations (1.1) and (1.2) it follows that the electric field transforms according to the relation $E' = E + w \times B$.

We shall make particular use of this transformation to relate the electric field in the laboratory frame $E$ to the electric field $E'$ in a frame of reference moving with the mean mass velocity of the fluid, as defined by equation (II 6.5). The electric fields in these two frames are then related by the equation

$$E' = E + u \times B. \quad (1.3)$$

Although we shall not use the transformation properties of the magnetic induction, we may note, for the sake of completeness, that in the non-relativistic approximation (see Hughes and Young, 1966, p. 143),

$$B' = B - u \times E/c^2. \quad (1.4)$$

**Exercise 1.1.** A current $I$ of 1 A flows along a copper wire whose cross section is 1 mm on a side. A magnetic induction field $B$ exists normal to the current flow. There is one conduction electron per atom in copper. Calculate the voltage across the two faces of the wire normal to $B$ and $I$. This phenomenon is known as the Hall effect, and the voltage is called the Hall voltage.