According to equation (5.11), the electrons can move a distance $\lambda_\text{D}$ in a
time $\omega_\text{pe}^{-1}$. Therefore, for any disturbance of lower frequency, the plasma
can respond sufficiently fast so as to maintain charge neutrality.

Exercise 5.1. Assuming that the ions and electrons can both move, calculate the oscillation frequency of the electron displacement relative to
that of the ions and compare the result with equation (5.5a).

6. ELECTROSTATIC PROBES

One of the earliest methods for obtaining spatially resolved measurements
of plasma properties was developed by Langmuir about 1924. A Langmuir
probe consists basically of a small electrode, frequently just a partially
exposed insulated wire, which is inserted into a plasma. A dc power supply
is attached to the probe and is usually arranged so that the potential of
the probe with respect to the plasma can be varied continuously over a
range of both negative and positive values. The current collected by the
probe is determined as a function of the biasing voltage, yielding a so-called
current-voltage (or $I$-$\phi$) characteristic. It is from the shape of this
characteristic that one attempts to derive information concerning plasma
properties.

Although it is relatively simple experimentally to obtain an $I$-$\phi$
characteristic, our present understanding of how probes behave is restricted
to rather special plasma conditions. The utility of an electrostatic probe
as a diagnostic device for other than these special kinds of plasmas is
dubious and is still the subject of active investigation. Useful reviews
concerning the theory and use of electrostatic probes have been written
by Chen (1965), by de Leeuw (1963), and by Schott (1968).

In this section we shall discuss several aspects of probe theory applicable
to plasma conditions similar to those prevailing in low-pressure discharges,
as studied by Langmuir. Important features of such discharges, as far as
probe theory is concerned, are that the plasma is nonmoving and the probe
is nonemitting. Also, the probe dimensions are much smaller than the mean
free paths of the plasma particles, so that the collection of charged particles
occurs under essentially collision-free conditions. In addition, the sheath
thickness is much less than the probe dimensions so that the sheath may be
treated as having plane symmetry. Our reason for discussing this theory
is that it is relatively simple and, at the same time, illustrates many of
the features common to current collection from a plasma. We shall discuss
some of the aspects involved in describing the effects of collisions in the
following section.

The general appearance of an $I$-$\phi$ characteristic is shown in Fig. 6. The
conventional current $I$ is defined as positive for flow from the electrode into the plasma, and is thus equivalent to regarding the net electron current as positive for flow from the plasma towards the electrode. The potential $\phi_p$ refers to the probe voltage relative to an arbitrary reference, often the anode or cathode for a discharge-produced plasma or a part of the plasma container. The net current collected by the probe is zero when the probe voltage is equal to the floating potential $\phi_f$. As discussed in Sec. 3, most of the electrons leaving the plasma are repelled by the probe at this condition in order that the separate electron and ion currents reaching the probe just balance. If $\phi_p$ is made negative with respect to $\phi_f$, more ions will reach the probe than electrons, resulting in a negative net electron current to the probe. If $\phi_p$ is sufficiently negative, all the electrons will be repelled, and the current collected by the probe will attain a nearly constant magnitude called the ion saturation current.

Figure 6. Schematic of a typical probe current-voltage characteristic. The inserts indicate the potential variation in the sheath for the two cases $\phi_p < \phi_s$ and $\phi_p > \phi_s$.
When the probe potential somewhat exceeds the floating potential, more electrons will be collected than ions, resulting in a positive value of $I$. As $\phi_p$ is increased, the electron current will continue to increase, resulting in an increasing value of $I$, until the probe voltage equals the plasma or space potential $\phi_s$. At this condition both the random electron and ion currents from the plasma reach the probe unimpeded. For all $\phi_p < \phi_s$, the probe is surrounded by a positive sheath as indicated in Fig. 6. When $\phi_p$ exceeds $\phi_s$ some of the ions are prevented from reaching the probe, and thus the net electron current to the probe is again increased, but the probe is now surrounded by a negative sheath. If $\phi_p$ is sufficiently positive, all the ions will be repelled and the probe will collect the electron saturation current. In practice, the probe current at either large negative or positive voltages does not saturate at a constant value, but continues to increase slowly in magnitude. This behavior is often attributable to an increasing sheath thickness, which results in an increasing effective current collection area for the probe.

In addition to the conditions previously discussed we shall assume, for simplicity, a single temperature. For $\phi_p \leq \phi_s$, equations (3.2) and (3.3) may be employed to obtain the expression

$$I = I_e^* \exp \left[ -\frac{e(\phi_s - \phi_p)}{kT} \right] - I_i^*.$$  \hspace{1cm} (6.1a)

For $\phi_p > \phi_s$,

$$I = I_e^* - I_i^* \exp \left[ -\frac{e(\phi_p - \phi_s)}{kT} \right].$$  \hspace{1cm} (6.1b)

Here

$$I_e^* = A e n \overline{C}_e /4,$$  \hspace{1cm} (6.2a)

$$I_i^* = A e n \overline{C}_i /4,$$  \hspace{1cm} (6.2b)

and $A$ denotes the current-collecting area of the probe. The theoretical current-voltage characteristic given by equations (6.1) is in accord with Fig. 6.

These results may be applied to obtain diagnostic information about plasma conditions in the following way. The ion saturation current $I_i^*$ can be measured directly by applying large negative bias voltages to the probe. The electron temperature can then be determined by fitting a straight line on semi-log graph paper to a plot of $\ln(I + I_i^*)$ vs. $\phi_p$, for values of $\phi_p$ in the transition regime (i.e., for $\phi_p$ near $\phi_s$). If the temperature is known, the electron number density may be calculated from the measured value of $I_i^*$. 


In principle, the relation
\[
\frac{kT}{e} = I_e^* \left[ \frac{d\phi_p}{dl} \right]_{l=0}
\]  
provides an alternative method for determining the temperature.

For charge neutrality to be maintained, an equal ion current must leave the plasma to balance the electron current drawn by the probe. In effect, the wall of the plasma container, or whatever other voltage reference is being used, acts as a second electrode to complete the circuit carrying the probe current. If the area of this effective electrode is too small, the value of the saturation current of the probe at large positive bias voltages will not be equal to the value \( I_e^* \) predicted by equation (6.1b). To see in detail how this comes about, and at the same time to discuss an alternative probe method proposed by Johnson and Malter (1950), let us consider next the two-electrode probe shown in Fig. 7.

Let us denote properties associated with the two electrodes by the subscripts 1 and 2, respectively, and let us suppose that \( A_2 \geq A_1 \). We may, if we wish, identify electrode number 1 with the single-electrode probe discussed previously. If \( A_2 \) is not too much greater than \( A_1 \), then as the applied voltage \( (\phi_1 - \phi_2) \) is increased, \( \phi_1 \) will approach a constant value while \( \phi_2 \) becomes more and more negative. The electron current into

\[ l = I_{e1} - I_{e2} \]

\[ l = I_{e1} - I_{e2} \]
electrode 1 will be limited by the maximum ion current that electrode 2 will accept. Provided the ion saturation current for electrode 2 satisfies the condition
\[ I_{e2}^* < I_{e1}^* - I_{i1}^* \]  
the potential \( \phi_1 \) will always remain less than the space potential \( \phi_s \). We shall limit our discussion to this case. The current-voltage characteristic for a double probe is shown schematically in Fig. 8.

To obtain a theoretical expression for the current-voltage characteristic, we note first that for a specified net electron current flow \( I \) to electrode 1, the potential \( \phi_1 \) of this electrode is determined by the relation [cf. equation (6.1a)]
\[ I = I_{e1}^* \exp \left[ \frac{-e(\phi_s - \phi_1)}{kT} \right] - I_{i1}^*. \]  
The potential \( \phi_2 \) of electrode 2 is then determined in terms of \( \phi_1 \) from the requirement that the current be continuous, which leads to the equation
\[ I_{e2}^* - I_{e2}^* \exp \left[ \frac{-e(\phi_s - \phi_2)}{kT} \right] = I_{e1}^* \exp \left[ \frac{-e(\phi_s - \phi_1)}{kT} \right] - I_{i1}^*. \]  
Solving equation (6.6) for \( \phi_2 - \phi_1 \), the applied potential difference \( \phi_2 - \phi_1 \) is given in terms of \( \phi_1 - \phi_s \) by the relation
\[ \exp \left[ \frac{e(\phi_1 - \phi_2)}{kT} \right] = \exp \left[ \frac{e(\phi_1 - \phi_s)}{kT} \right] \exp \left[ \frac{e(\phi_s - \phi_2)}{kT} \right] \]
\[ = \frac{I_{e1}^* + I_{e2}^*}{I_{e2}^*} \frac{I_{e1}^*}{I_{e2}^*} \exp \left[ \frac{e(\phi_1 - \phi_s)}{kT} \right] \]
From this equation \((\phi_1 - \phi_2)\) may be obtained in terms of \((\phi_1 - \phi_2)\), and the result substituted into equations (6.5). One then obtains for the current-voltage characteristic, the relation

\[
I = \frac{I_{n1}^\ast (I_{n2}^\ast /I_{n2}^\ast) \exp[e(\phi_1 - \phi_2)/kT] - I_{n1}^\ast}{(I_{n1}^\ast /I_{n2}^\ast) \exp[e(\phi_1 - \phi_2)/kT] + 1}.
\] (6.7)

For \((\phi_1 - \phi_2) \to -\infty\) we recover from equation (6.7) the same result as for a single probe, that \(I \to -I_{n1}^\ast\). However, for \((\phi_1 - \phi_2) \to \infty\) equation (6.7) shows that \(I \to I_{n1}^\ast\), in contrast with the result \(I \to I_{n1}^\ast\) for a single probe. This behavior illustrates the need for caution in interpreting electron saturation current data obtained with single probes.

In the limit \(A_2 \gg A_1\), and for currents \(I\) in the ion saturation and transition regimes for electrode 1, one can show that the current-voltage characteristic for a double probe is identical with that for a single probe. For the conditions stated, the current collected by electrode 2 will be negligible in comparison with \(I_{n2}^\ast\), so that \(\phi_2 \approx \phi_f\) and

\[
I_{n2}^\ast \approx I_{n2}^\ast \exp[-e(\phi_2 - \phi_f)/kT].
\]

Making these substitutions in equation (6.7) and neglecting the first term in the denominator, which is justified since for the conditions stipulated \(e(\phi_1 - \phi_2)/kT \ll 1\), we obtain

\[
I \approx I_{n1}^\ast \exp[-e(\phi_2 - \phi_1)/kT] - I_{n1}^\ast.
\]

Comparison of this result with equation (6.1a) for a single probe shows that the characteristics are indeed identical under the conditions stated.

For diagnostic applications, corresponding to equation (6.3) for a single probe, one may show from equation (6.7) that for a double probe

\[
\frac{kT}{e} = \frac{I_{n1}^\ast}{1 + (A_1/A_2) \left[ \frac{d(\phi_1 - \phi_2)}{dI} \right]_{I=0}}.
\] (6.8)

Double probes are usually used with \(A_2 = A_1\), in which case equation (6.7) can be written in the simplified form

\[
I = I_{n1}^\ast \tanh[e(\phi_1 - \phi_2)/2kT].
\]

To conclude this section, it should be pointed out that the measured potential of an electrode differs from the effective potential at the surface by a so-called contact potential, or work function. As long as the contact potential is uniform over the probe surface and does not change with time, the shapes of the probe characteristics discussed above remain unaltered. Under some experimental conditions, variations in contact potential can be important, and steps must be taken to minimize their effects.
Exercise 6.1. Derive the relations (6.3) and (6.8).

Exercise 6.2. Discuss the theory of double probe characteristics when the condition (6.4) does not apply.

7. AMBIPOlar DIFFUSION

For collision-dominated partially ionized gases, the diffusion of charged particles is determined not only by the electric field, as discussed in Sec. II 13, but also by gradients in plasma properties. Thus, in place of equation (II 13.6a), the electron diffusion velocity may be written

$$U_e = -\mu_e \left( E + \frac{\nabla p_e}{en_e} \right).$$  \hspace{1cm} (7.1a)

Here $p_e = n_e k T_e$ is the electron pressure, and $\mu_e$ is the electron mobility. As shown by equation (II 13.6b), $\mu_e \approx e/m_e \bar{v}_e H$. (In this section we shall take the magnetic field as zero, and thus $E' = E$. The effects of a magnetic field are discussed in Sec. IV 8.) Equation (7.1a) is obtained in Sec. IV 8 using an approach based on fluid conservation equations, and it is derived more rigorously in Chapter VIII on the basis of kinetic theory. The conditions for the validity of this expression are discussed in detail in Chapter VIII. Briefly, the use of equation (7.1a) requires that the effects of thermal diffusion be small and that the electron velocity distribution function be approximately Maxwellian. The expression for $U_e$ is often written in the more general form

$$U_e = -\mu_e E - D_e \frac{\nabla p_e}{p_e},$$  \hspace{1cm} (7.1b)

where $D_e$ is the electron diffusion coefficient. For the conditions of validity of equation (7.1a),

$$D_e/\mu_e = k T_e/e.$$  \hspace{1cm} (7.2)

This result is frequently referred to as the Einstein relation.

The ion diffusion velocity for a weakly ionized gas, and for approximately uniform total pressure, may be written

$$U_i = \mu_i \left( E - \frac{\nabla p_i}{en_i} \right).$$  \hspace{1cm} (7.3a)

Here $p_i = n_i k T$ is the ion pressure, $T$ is the heavy particle temperature, and $\mu_i$ is the ion mobility. For a three-species gas $\mu_i \approx e/m_i \bar{v}_i$ in accordance with equation (II 13.13). Equation (7.3a) may be derived on the basis of fluid conservation equations, as described in Exercise IV 8.3,